8.7 L'Hopital's Rule

Obj: Recognize limits as indeterminate; Use L'hopital's Rule to find limits

Limit Review:

Consider the limit: $\lim_{x\to 2}\frac{x^2-4}{x-2}$

Indeterminate forms:

L'Hopital's Rule:

Example 1: $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ Check to see if indeterminate first

Example 2: $\lim_{x\to 0}\frac{1-\cos x}{x+x^2}$

On the other hand, you can apply L'Hôpital's rule as many times as necessary as long as the fraction is still indeterminate:

Example 3: $\lim_{x\to 0}\frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2}$

You can also evaluate one sided limits. Many times your answers will be ∞ or $-\infty$.(if C/0) because you have vertical asymptote.

Example 4:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}$$

$$\lim_{x\to 0^-} \frac{\sin x}{x^2}$$

Finding Horizontal asymptotes:

Ex. $\lim_{x\to\infty} \frac{\ln x}{x}$

$$\lim_{x\to -\infty} \frac{x^2}{e^{-x}}$$

You try: $\lim_{x\to\infty} \frac{e^x}{x}$

Other indeterminate forms:

$$\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right)$$

$$\lim_{x\to\infty} e^{-x}\sqrt{x}$$

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

Yet more…

$$\lim_{x\to\infty}(1+\frac{1}{x})^{x}$$

$$\lim_{x\to\infty} x^{1/x}$$

$$\lim_{x\to 0^{+}}(\sin x)^{x}$$